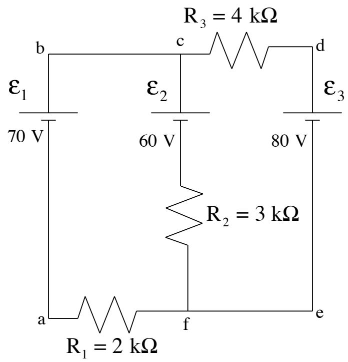
Problem 18.23

What is the current in each branch?

How many nodes:

How many loops:

How many independent equations needed?



18.23) What is the current in each branch?

Node c:

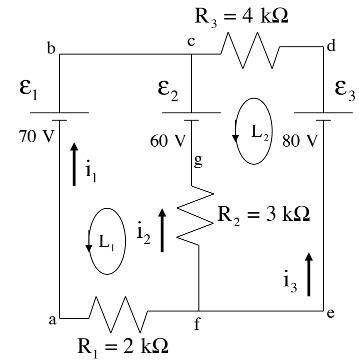
$$i_1 + i_2 + i_3 = 0$$

Loop 1 (starting at *a* and moving c.c.):

$$+R_1 i_1 - R_2 i_2 + \varepsilon_2 - \varepsilon_1 = 0$$

Loop 2 (starting at *g* and moving c.c.):

$$R_2 i_2 + \varepsilon_3 - R_3 i_3 - \varepsilon_2 = 0$$



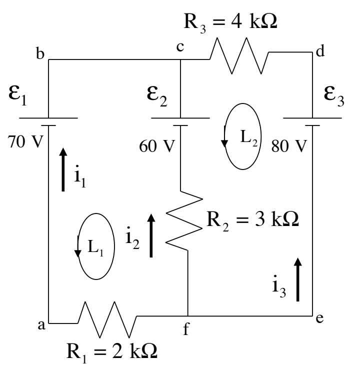
Problem 18.23

What is the current in each branch?

Node c:

Loop 1:

Loop 2:



Rearranging into matrix format:

L1: $+R_1 i_1 - R_2 i_2 + 0 i_3 = -\varepsilon_2 + \varepsilon_1$

L2: $0 i_1 + R_2 i_2 - R_3 i_3 = \varepsilon_2 - \varepsilon_3$

Nc: $1 i_1 + 1 i_2 + 1 i_3 = 0$

Putting in the numbers:

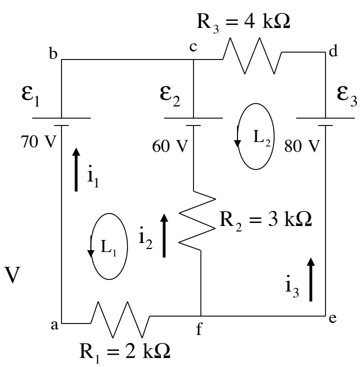
L1: $(2000\Omega) i_1 - (3000\Omega) i_2 + 0 i_3 = -60\text{ V} + 70\text{ V}$

$\Rightarrow 2000 i_1 - 3000 i_2 = 10$

L2: $0 i_1 + (3000\Omega) i_2 - (4000\Omega) i_3 = 60\text{ V} - 80\text{ V}$

$\Rightarrow 3000 i_2 - 4000 i_3 = -20$

Nc: $1 i_1 + 1 i_2 + 1 i_3 = 0$



$$2000i_1 - 3000i_2 = 10$$
$$3000i_2 - 4000i_3 = -20$$
$$1i_1 + 1i_2 + 1i_3 = 0$$

Putting this information into matrix form, we take the coefficients of each of the current-variables and place them in a matrix such that each column is devoted to a particular unknown current (that is, the first column would be devoted to the unknown current i_1, etc.).

i_1 column	i_2 column	i_3 column		constants column
2000	–3000	0		10
0	3000	–4000	=	–20
1	1	1		0

$$\begin{vmatrix} 2000 & -3000 & 0 \\ 0 & 3000 & -4000 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 10 \\ -20 \\ 0 \end{vmatrix}$$

As was explained in class, the solution to any one individual variable current (say i_1) can be determined to dividing the evaluation of the determinant (that's the matrix above and to the left) into the evaluation of the determinant with the i_1 column replaced by the constants column. In other words, it looks like the set-up shown to the right where I've used red numbers to highlight the substitution:

$$i_1 = \frac{\begin{vmatrix} 10 & -3000 & 0 \\ -20 & 3000 & -4000 \\ 0 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 2000 & -3000 & 0 \\ 0 & 3000 & -4000 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$2000i_1 - 3000i_2 = 10$$
$$3000i_2 - 4000i_3 = -20$$
$$1i_1 + 1i_2 + 1i_3 = 0$$

I like to do the same evaluation over and over again, so I reproduce the first and second column to the right of each matrix so I can execute over and over again the upper-left times bottom-right minus upper-right times bottom-left operation highlighted below with circles and parentheses shown below:

$$i_1 = \frac{\begin{vmatrix} 10 & -3000 & 0 \\ -20 & 3000 & -4000 \\ 0 & 1 & 1 \end{vmatrix} \begin{matrix} 10 & -3000 \\ -20 & 3000 \\ 0 & 1 \end{matrix}}{\begin{vmatrix} 2000 & -3000 & 0 \\ 0 & 3000 & -4000 \\ 1 & 1 & 1 \end{vmatrix} \begin{matrix} 2000 & -3000 \\ 0 & 3000 \\ 1 & 1 \end{matrix}}$$

$$i_1 = \frac{(10)[(3000)(1)-(-4000)(1)]+(-3000)[(-4000)(0)-(-20)(1)]+(0)[(-20)(1)-(3000)(0)]}{(2000)[(3000)(1)-(-4000)(1)]+(-3000)[(-4000)(1)-(0)(1)]+(0)[(0)(1)-(3000)(1)]}$$

$$= \frac{70{,}000-60{,}000+0}{14{,}000{,}000+12{,}000{,}000+0} = \frac{10{,}000}{26{,}000{,}000}$$

$$= 3.85\text{x}10^{-4} \text{ amps}$$

$$2000i_1 - 3000i_2 = 10$$
$$3000i_2 - 4000i_3 = -20$$
$$1i_1 + 1i_2 + 1i_3 = 0$$

Continuing:

$$i_2 = \frac{\begin{vmatrix} 2000 & 10 & 0 \\ 0 & -20 & -4000 \\ 1 & 0 & 1 \end{vmatrix} \begin{matrix} 2000 & 10 \\ 0 & -20 \\ 1 & 0 \end{matrix}}{\begin{vmatrix} 2000 & -3000 & 0 \\ 0 & 3000 & -4000 \\ 1 & 1 & 1 \end{vmatrix} \begin{matrix} 2000 & -3000 \\ 0 & 3000 \\ 1 & 1 \end{matrix}}$$

$$i_2 = \frac{(2000)[(-20)(1)-(-4000)(0)]+(10)[(-4000)(1)-(0)(1)]+(0)[(0)(0)-(-20)(1)]}{(2000)[(3000)(1)-(-4000)(1)]+(-3000)[(-4000)(1)-(0)(1)]+(0)[(0)(1)-(3000)(1)]}$$

$$= \frac{-40{,}000-40{,}000+0}{14{,}000{,}000+12{,}000{,}000+0} = \frac{-80{,}000}{26{,}000{,}000}$$

$$= -3.08\text{x}10^{-3} \text{ amps}$$

NOTE: What does the negative sign mean? It means simply that we assumed the wrong current direction for the current when starting the problem. It shouldn't be surprising that we've found one of these. There is no way the currents included in Kirchoff's Second Law (sum of the currents into a node, etc.) could all three be positive, as originally assumed.

$$2000i_1 - 3000i_2 = 10$$
$$3000i_2 - 4000i_3 = -20$$
$$1i_1 + 1i_2 + 1i_3 = 0$$

Continuing:

$$i_3 = \frac{\begin{vmatrix} 2000 & -3000 & 10 \\ 0 & 3000 & -20 \\ 1 & 1 & 0 \end{vmatrix} \begin{matrix} 2000 & -3000 \\ 0 & 3000 \\ 1 & 1 \end{matrix}}{\begin{vmatrix} 2000 & -3000 & 0 \\ 0 & 3000 & -4000 \\ 1 & 1 & 1 \end{vmatrix} \begin{matrix} 2000 & -3000 \\ 0 & 3000 \\ 1 & 1 \end{matrix}}$$

$$i_3 = \frac{(2000)[(3000)(0)-(-20)(1)]+(-3000)[(-20)(1)-(0)(0)]+(10)[(0)(1)-(3000)(1)]}{(2000)[(3000)(1)-(-4000)(1)]+(-3000)[(-4000)(1)-(0)(1)]+(0)[(0)(1)-(3000)(1)]}$$

$$= \frac{40{,}000+60{,}000-30{,}000}{14{,}000{,}000+12{,}000{,}000+0} = \frac{70{,}000}{26{,}000{,}000}$$

$$= 2.69\text{x}10^{-3} \text{ amps}$$

Of course, you could have solved the problem long-hand. Doing so yields:

From Loop 1:

$$2000i_1 - 3000i_2 = 10$$
$$\Rightarrow \quad i_1 = 1.5i_2 + .005$$

From Loop 2:

$$3000i_2 - 4000i_3 = -20$$
$$\Rightarrow \quad i_3 = .75i_2 + .005$$

Putting it all together in equation from Node c:

$$i_1 \quad + \quad i_2 \quad + \quad i_3 \quad = 0$$
$$\Rightarrow \left(1.5i_2 + .005\right) + i_2 + \left(.75i_2 + .005\right) = 0$$
$$\Rightarrow i_2 = -3.08\text{x}10^{-3}\ \text{A}$$
$$\Rightarrow i_1 = 1.5i_2 + .005 = 1.5\left(-3.08\text{x}10^{-3}\right) + .005 = 3.85\text{x}10^{-4}\ \text{A}$$
$$\Rightarrow i_3 = .75i_2 + .005 = .75\left(-3.08\text{x}10^{-3}\right) + .005 = 2.69\text{x}10^{-3}\ \text{A}$$

OR, if you happen to be a wizard with a TI (whatever), you could do the following:

1.) Hit MATRIX, then EDIT, then select *matrix A* to be a 3x3 matrix and put in:

$$\begin{vmatrix} 2000 & -3000 & 0 \\ 0 & 3000 & -4000 \\ 1 & 1 & 1 \end{vmatrix}$$

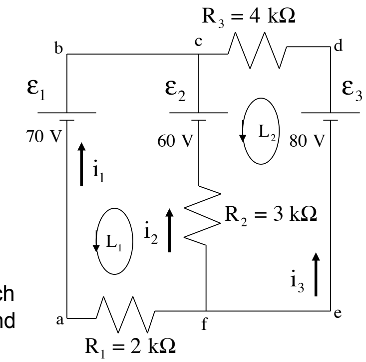
2.) Hit QUIT, then MATRIX again, then EDIT, then select *matrix B* to be a 3x1 and put in:

$$\begin{vmatrix} 10 \\ -20 \\ 0 \end{vmatrix}$$

3.) Hit QUIT, then (A) . . . this will call up *matrix A* . . . , then x^{-1} . . . this will give the inverse of matrix A . . . , then the times sign, then hit (B) . . . this will call up *matrix B* . . . , then ENTER.

What you will end up with will be SOUGHT AFTER "i" VALUES!!! $\begin{vmatrix} 3.85\text{x}10^{-4} \\ 3.08\text{x}10^{-3} \\ 2.69\text{x}10^{-3} \end{vmatrix}$, which it to say THE THREE. Nifty, eh?

b.) As for the voltage across *c* and *f*, simply following the voltage changes (keeping track of positive and negative signs) from *c* to *f*. Using the current directions as originally defined, we get:



$$\Delta V_{cf} = -\varepsilon_2 + i_2R_2$$
$$= -(60\ \text{V}) + \left(-3.08\text{x}10^{-3}\ \text{A}\right)(3000\ \Omega)$$
$$= -69.24\ \text{V}$$

To check this, we can use the same approach to determine the voltage change across *b* and *f* going counterclockwise. (That number should be the same as the voltage change across *c* and *f*.) Doing so yields:

$$\Delta V_{bf} = -\varepsilon_1 + i_1R_1$$
$$= -(70\ \text{V}) + \left(.38\text{x}10^{-3}\ \text{A}\right)(2000\ \Omega)$$
$$= -69.24\ \text{V}$$

Great jumping hazzahs.
It works!